Springer

KSME

Journal of Mechanical Science and Technology 24 (6) (2010) 1203~1210
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-010-0333-4

Mathematical model for determination of strand twist angle and diameter in stranded-wire helical springs †

Shilong Wang*, Song Lei, Jie Zhou and Hong Xiao

The State Key Laboratory of Mechanical Transmission, Chongqing University, Chongqinq 400044, China

(Manuscript Received January 8, 2009; Revised December 15, 2009; Accepted March 8, 2010)

Abstract

The twist angle of strands has a significant effect on the performance of stranded-wire helical springs. This paper proposes two models (a precise mathematical model and an approximate mathematical model) for calculation of the twist angle and the diameter of strands formed of an arbitrary number of wires. These two parameters can be derived from the screw pitch of strands and the diameter and number of their wires, regardless of the model used. In comparative and analytical studies, it was determined that for strands with the same number of wires, the larger the ratio of the screw pitch to the diameter of the wires is, the smaller the relative errors of the two models are. Only when the foresaid ratio is two times larger than the number of wires, the results of the approximate mathematical model can be acceptable. In other cases, the approximate mathematical model cannot be used. Finally, application software was developed for direct precise-mathematical-model calculation of the twist angle and diameter of strands formed of arbitrary wires.

Keywords: Stranded-wire helical spring; Strand; Twist angle; Diameter; Design

1. Introduction

A stranded-wire helical spring, first observed in Russian machine guns [1, 2], is a unique cylindrically helical spring. It is reeled by a strand that is formed of 2~7 wires. The diameter of the wires is 0.5~3.0 mm, and their material is common carbon steel.

It has been known for some time that stranded-wire helical springs, as compared with conventional single helical springs, exhibit more damping and have longer fatigue lives in certain dynamic applications [3-5]. They have been used successfully in machine guns, small weapons, and stapling guns to dampen the high-velocity displacement of the coils [6, 7].

For best stranded spring performance, it is essential that strands be designed and manufactured accurately [8]. As shown in Fig. 1, the twist angle of a strand, β, has a significant impact on stranded spring performance. In the design of stranded-wire helical springs, the twist angle β and the diameter d_c are calculated according to the screw pitch of strands S, the diameter of wires d and the number of wires N.

In Zhang, Liu and Wang's *Spring Handbook* [9], and in the paper by Maolin Wang [10], a method of calculating strand diameter whereby the minimum distance of adjacent wires is to be equal to the diameter of wires was offered. In this method, the twist angle of strands first is assumed. Then, this angle is used to obtain the strand diameter in reference to a lookup table. However, as a matter of fact, the twist angle is determined via the screw pitch of strands and the diameter of wires. Therefore, this method does not solve the problem of the calculation of β and d_c suitably.

In the Handbook *of Spring Design, New Technology of Manufacturing Process and Quality Control*, Dongwen Zhang [11] proposed a calculation method in which the cross-section of a strand is composed of ellipses in contact with each other. However, it offered approximate calculation of strands of only three or four wires; for cases in which the number of wires exceeds four, the method is infeasible. Under some circumstances, the values of parameters can be acquired using a method of interpolation, mentioned in papers by Costello [1, 3], Clark [2] and Sathikh [12]. However, these papers offer no explicit calculation process.

Recently, Shilong Wang and Jie Zhou [13] formulated a mathematical model for stranded-wire helical spring tension during reeling. It is well known that in deriving such a model, the most important parameter is the center distribution radius of strands r.

From the structural dimensions of a strand (see Fig. 1), two expressions (see Eqs. (1-2)) can be obtained:

† This paper was recommended for publication in revised form by Associate Editor Tae Hee Lee

*Corresponding author. Tel.: +86 23 6510 3567, Fax.: +86 23 6510 6436 (801)
E-mail address: slwang@cqu.edu.cn

© KSME & Springer 2010

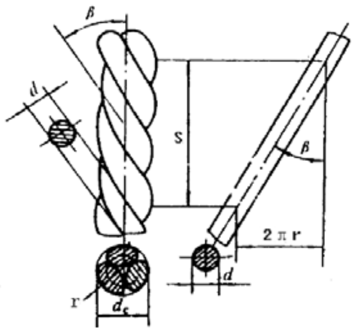
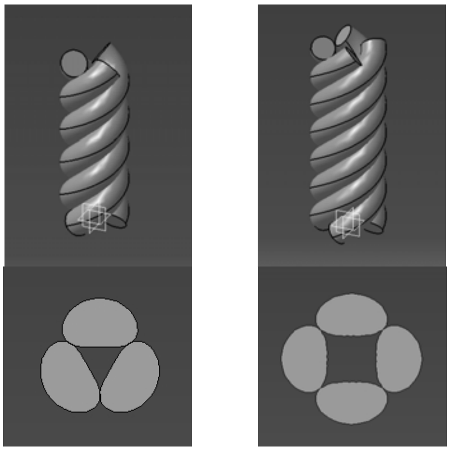


Fig. 1. Structural dimensions of strand.



(a) (b)

Fig. 2. Strands and their cross-sections: (a) three strands; (b) four strands.

$$d_c = 2r + d \tag{1}$$

$$\tan\beta = \frac{2\pi r}{S} \tag{2}$$

In fact, the cross-section of strands is composed of irregular figures, not circles or ellipses, in contact with each other (see Fig. 2). The purpose of the present study was to test two strand cross-section models, a precise mathematical model and an approximate mathematical model. Specifically, the two methods' calculations of the twist angle and diameter of strands of arbitrary wires were compared and analyzed. Based on the results, the relationships among S, d, β and N were obtained. Finally, application software was developed for direct precise-mathematical-model calculation of the twist angle and diameter of strands of arbitrary wires.

2. Mathematical model of cross-section of strands

2.1 Precise mathematical model of cross-section of strands

Fig. 2 depicts helical-spring strands formed of either three or four twisted wires, along with their cross-sections.

For precise mathematical modeling of the irregular cross-section of strands, two Cartesian coordinate systems were derived according to the forming process for an arbitrary-wire-twisted strand (see Fig. 3). The first coordinate system is

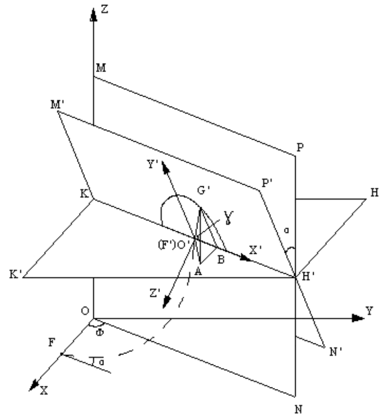


Fig. 3. Forming process of strand.

based on the original point O (confirmed as σ = {O: X, Y, Z}). FF' is the center line of an optional wire in the strand. OF is equal to the center distribution radius of the strand. The plane M'P'KH' is the normal plane of the center line FF' at point F'. The point F' is coincident with the original point O' in the second coordinate system; the cross-section of the wire is a circular wire section in the normal plane M'P'KH'. The second coordinate system was confirmed as σ' (σ' = {O': X', Y', Z'}). By means of a series of translations and rotations, σ can be transformed to σ'.

According to the affine transformation of the Cartesian coordinate systems, there is a relation between σ and σ' [14] (see Eqs. (3-4)). That is,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = A \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \tag{3}$$

or

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = A' \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} \tag{4}$$

A, an orthogonal matrix, is called the transformation matrix. According to the definition of an orthogonal matrix, the relation $A^{-1} = A'$ can be obtained. The translation coordinates (X_0, Y_0, Z_0) are the coordinates of point O' in the coordinate system σ. In the mathematical model shown in Fig. 3, (X_0, Y_0, Z_0) is equal to ($r\cos\phi, r\sin\phi, S\phi/2\pi$), via the equation of the spiral line.

A' is also the rotation matrix. In order to impart a specific orientation to an object, it is subjected to a sequence of three rotations determined by Euler angles. This is to say, the rotation matrix can be decomposed into three elemental rotations. Assuming that the intersection of the OXY and O'X'Y' coordinate planes is a line of nodes (the $\triangle$-axis), ϕ is the angle between the X-axis and the $\triangle$-axis, θ is the angle between the Z-axis and the Z'-axis, and φ is the angle between the $\triangle$-axis and the X'-axis. The ranges of ϕ and φ are de-

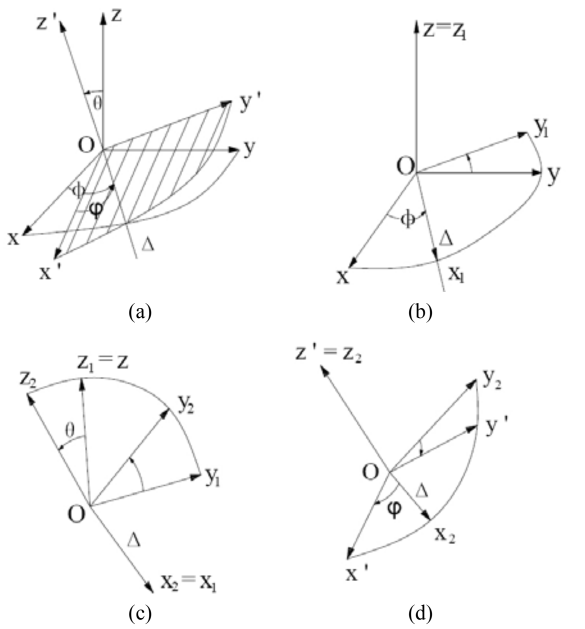


Fig. 4. Three rotations described by Euler angles: (a) three Euler angles; (b) rotation around Z-axis; (c) rotation around intermediate △-axis; (d) rotation around Z'-axis.

fined as modulo 2π radians. A valid range could be $[-\pi,\pi]$. The θ range is a modulo π radian, for example, $[0,\pi]$ or $[-\pi/2,\pi/2]$ (see Fig. 4).

According to the above description [14],

$$A' = A'_3 A'_2 A'_1 = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

where matrix A'_1 represents a rotation around the Z-axis of the original reference frame σ, matrix A'_2 is a rotation around an intermediate △-axis (i.e. the line of nodes), and matrix A'_3 is a rotation around the Z'-axis of the final reference frame (see Eq. (5)).

Carrying out the matrix multiplication, the result is

$$A' = \begin{pmatrix} \cos\varphi\cos\phi - \cos\theta\sin\varphi\sin\phi & \cos\varphi\sin\phi + \cos\theta\sin\varphi\cos\phi & \sin\theta\sin\varphi \\ -\sin\varphi\cos\phi - \cos\theta\cos\varphi\sin\phi & -\sin\varphi\sin\phi + \cos\theta\cos\varphi\cos\phi & \sin\theta\cos\varphi \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{pmatrix} \tag{6}$$

Because the X'-axis is parallel to the OXY coordinate plane, in the mathematical model (see Fig. 3), $\varphi = 0$ and $\theta = \pi/2 + \alpha$ are obtained. Substituting these values, the simplified version of Eq. (6) is

$$A' = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ \sin\alpha\sin\phi & -\sin\alpha\cos\phi & \cos\alpha \\ \cos\alpha\sin\phi & -\cos\alpha\cos\phi & -\sin\alpha \end{pmatrix} \tag{7}$$

Combining Eq. (4) with Eq. (7), yields

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = A' \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ \sin\alpha\sin\phi & -\sin\alpha\cos\phi & \cos\alpha \\ \cos\alpha\sin\phi & -\cos\alpha\cos\phi & -\sin\alpha \end{pmatrix} \begin{pmatrix} X - r\cos\phi \\ Y - r\sin\phi \\ Z - \frac{S}{2\pi}\phi \end{pmatrix} \tag{8}$$

As we have known, in the coordinate system σ', the normal cross-section of a wire is a circle. Therefore, the basic equation of a circle can be used to represent the shape of the normal cross-section (see Fig. 3):

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} \frac{d}{2}\cos\lambda \\ \frac{d}{2}\sin\lambda \\ 0 \end{pmatrix} \tag{9}$$

where λ is an intermediate variable of Eq. (9). Combining Eq. (8) with Eq. (9), the result is

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} (r + \frac{d}{2}\cos\lambda)\cos\phi + \frac{d}{2}\sin\alpha\sin\lambda\sin\phi \\ (r + \frac{d}{2}\cos\lambda)\sin\phi - \frac{d}{2}\sin\alpha\sin\lambda\cos\phi \\ \frac{d}{2}\cos\alpha\sin\lambda + \frac{S}{2\pi}\phi \end{pmatrix} \tag{10}$$

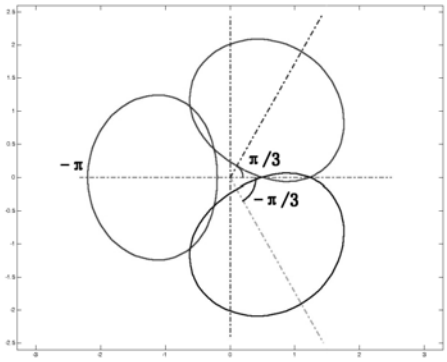
This is the circle equation (see Eq. (9)) expressed for the coordinate system σ. Then, for the final cross-section equation of a wire in the coordinate system σ, where Z is zero, Eq. (10) becomes

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} (r + \frac{d}{2}\cos\lambda)\cos\phi + \frac{d}{2}\sin\alpha\sin\lambda\sin\phi \\ (r + \frac{d}{2}\cos\lambda)\sin\phi - \frac{d}{2}\sin\alpha\sin\lambda\cos\phi \end{pmatrix}$$

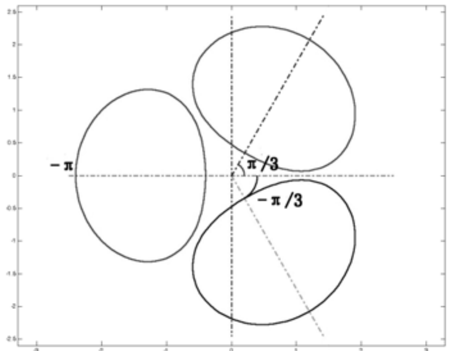
where

$$\begin{cases} \phi = -\dfrac{\pi d}{S}\cos\alpha\sin\lambda \\ \cos\alpha = \sin\beta = \dfrac{2\pi r}{\sqrt{(2\pi r)^2 + S^2}} \\ \sin\alpha = \cos\beta = \dfrac{S}{\sqrt{(2\pi r)^2 + S^2}} \end{cases} \tag{11}$$

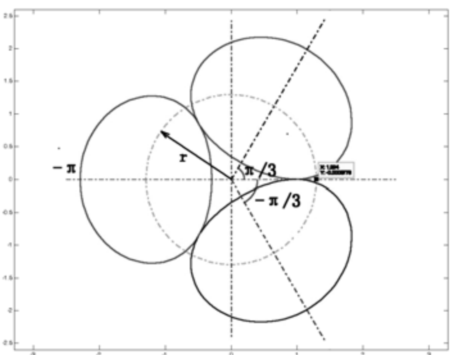
The phase angle of the cross-section Eq. (11) is zero. The difference in the phase angles of adjacent wires is $2\pi/N$. Therefore, by means simply of changing the phase angle, it is



(a)



(b)



(c)

Fig. 5. Cross-sections of three wires: (a) intersection of cross-sections; (b) separation of cross-sections; (c) tangency of cross-sections.

very convenient to obtain the equations for other cross-sections of strands.

As shown in Fig. 5, the strand is formed of three twisted wires. The positions of the cross-sections represent the fact that the phase angles are separate: $-\pi/3, -\pi, \pi/3$. With the values of the parameters, S, d, r, the cross-sections can be obtained. Fig. 5 illustrates the three cases of intersection, separation and tangency.

Usually, the values of S and d are already known. The center distribution radius r is determined so that the cross-sections of the strand are tangential to each other (see Fig. 5(c)). However, it is impossible to obtain the analytical solution of r from the known S and d, because Eq. (11) is transcendental. Therefore, in order to find the final solution, the numerical method has to be used.

Take a wire for which the phase angle is $-\pi/3$; the cross-section equation therefore is

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} (r+\frac{d}{2}\cos\lambda)\cos(\phi-\frac{\pi}{3})+\frac{d}{2}\sin\alpha\sin\lambda\sin(\phi-\frac{\pi}{3}) \\ (r+\frac{d}{2}\cos\lambda)\sin(\phi-\frac{\pi}{3})-\frac{d}{2}\sin\alpha\sin\lambda\cos(\phi-\frac{\pi}{3}) \end{pmatrix} \tag{12}$$

Here, the bisection method is used to get the numerical solution of r. By observation, the maximum value of Eq. (12) is zero when the cross-sections are tangential each other (see Fig. 5(c)). In the case of intersection (as confirmed by a center distribution radius r_a) and separation (a center distribution radius r_b), the maximum values are respectively larger (see Fig. 5(a)) and smaller (see Fig. 5(b)) than zero. The maximum values are indicated by MAX (r_a) and MAX (r_b), respectively. Given an absolute error, the bisection method converges to a root of the maximum value very close to zero. And the value of the root is the numerical solution of r.

Given d =2 mm, S =10 mm, $r_a = d/10$ =0.2 mm and $r_b = 10d$ = 20 mm, the center distribution radius of the strand r = 1.2944 mm, the diameter of the strand $d_c = 2r + d = 2\times1.2944 + 2 = 4.5888$ mm, and the twist angle of the strand $\beta = \arctan(2\pi r/S) = \arctan(2\times3.14\times1.2944/10) = 39.1219°$.

2.2 Approximate mathematical model of cross-section of strands

In the preceding section, a precise mathematical model of the cross-section of a strand was proposed. However, this model is very complex. In order to obtain a precise center distribution radius solution, a program code must be written. In this section, for the purposes of simplifying the calculation of the center distribution radius, an approximate mathematical model is presented.

Assume that the cross-section of a strand is composed of ellipses in contact with each other. The tangent lines at the contact points of the wires pass through the center of the strand. The center distribution circle passes through every tangent point (Actually, the center distribution circle in the precise mathematical model does not pass through every tangent point, as shown in Fig. 5(c). Here, an assumption is made in order to simplify the calculation.). Establish a plane coordinate system of which the original point is the center of a selected wire, as shown in Fig. 6.

No matter how many wires are in the same layer, the slope coefficient of the tangent line O'M can be expressed with the equation

$$K = \tan(90° - 180°/N) \tag{13}$$

Specifically, for a strand of three and four wires, K is

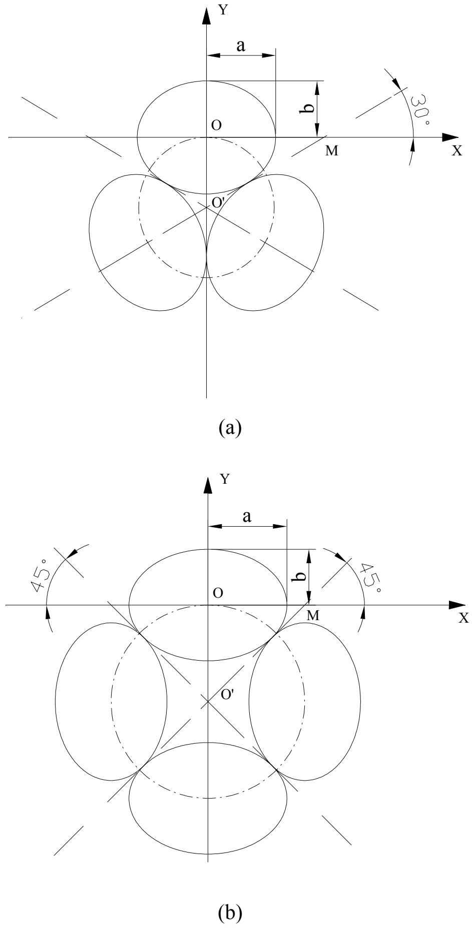


Fig. 6. Approximate cross-section of strand: (a) formed of three wires; (b) formed of four wires.

equal to $\sqrt{3}/3$ and 1, respectively.

The major radius of the ellipse, of which the center is the original point of the plane coordinate system, is expressed by

$$a = \frac{d}{2\cos\beta} \tag{14}$$

The minor radius is expressed by

$$b = d/2 \tag{15}$$

Combining Eq. (14) and Eq. (15), the cross-section equation of the ellipse is

$$(\cos^2\beta)x^2 + y^2 = (\frac{d}{2})^2 \tag{16}$$

The tangent line O'M passes through the center of the center distribution circle, the equation of the tangent line O'M can be expressed by

$$y = Kx - r \tag{17}$$

Combining Eq. (16) and Eq. (17) yields the expression

$$(\cos^2\beta + K^2)x^2 - 2Krx + (r^2 - \frac{d^2}{4}) = 0 \tag{18}$$

On account of the tangency between O'M and the ellipse, there is a double root of Eq. (18). As such, it is easy to obtain the expression of the center distribution radius by

$$r = \frac{d}{2}\sqrt{1 + \frac{K^2}{\cos^2\beta}} \tag{19}$$

In accordance with $\cos\beta = S/\sqrt{(2\pi r)^2 + S^2}$, the final expression of r is

$$r = \frac{dS}{2}\sqrt{\frac{K^2+1}{S^2 - \pi^2 K^2 d^2}}$$

where

$$K = \tan(90° - \frac{180°}{N}) \tag{20}$$

Take the cross-section of a strand formed of three twisted wires; for the same values $d = 2$ mm, $S = 10$ mm, $N = 3$, the results are $K = 0.5774$, $r = 1.2391$ mm, $d_c = 2r + d = 2\times1.2391 + 2 = 4.4782$ mm, $\beta = \arctan(2\pi r/S) = \arctan(2\times3.14\times1.2391/10) = 37.9026°$.

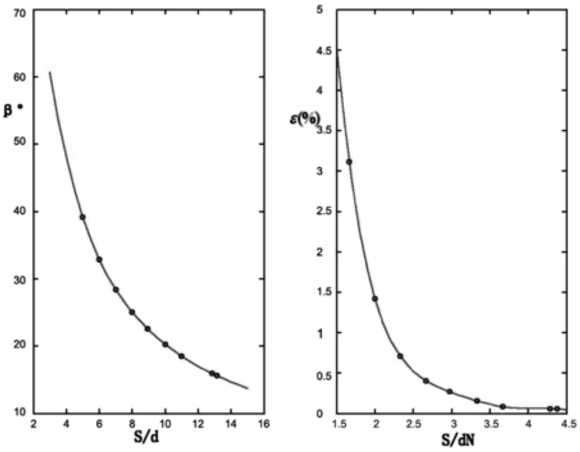
3. Comparison and analysis

The results for the approximate model are smaller than those of the precise model. In order to verify the accuracy of the approximate mathematical model, the twist angles and diameters of some strands to be used for stranded-wire helical springs in automatic weapons were calculated using the two methods. Additionally, the relative errors were obtained by introducing the S/d ratio as an important parameter (see Table 1).

Regardless of the mathematical model used, precise or approximate, or the number of strand wires, the twist angle and strand diameter could be obtained directly from the strand screw pitch, the wire diameter and the wire number. An analysis of the data showed that the larger the S/d ratio is, the smaller the relative errors are, for the same number of wires in the strand. With an equal S/d ratio, a greater number of wires results in larger relative errors. There is no change in the relative errors and twist angles with the same ratio S/d and number of wires in the strand, regardless of the specific values of d and S. Fig. 7 illustrates the relationships between the twist angle β and the S/d ratio, and between the relative error of the two models and the S/dN ratio, for a strand formed of three wires.

Table 1. Comparison of values for two models.

Weapon Names	N	d/mm	S/mm	r/mm			d_s /mm			β /°			S/d
				I	II	ε (%)	I	II	ε (%)	I	II	ε (%)	
M60 7.62 mm, Machine Gun, America	3	1.0	8	0.5955	0.5928	0.4601	2.1911	2.1856	0.2501	25.0667	24.9655	0.4039	8
14.5 mm, Antiair-craft Machine Gun, 1956	3	2.8	25	1.6559	1.6510	0.2959	6.1118	6.1020	0.1603	22.5958	22.5357	0.2662	8.93
12.7 mm, Aircraft Machine Gun, 1959	3	2.3	23	1.3526	1.3503	0.1692	5.0052	5.0006	0.0914	20.2794	20.2479	0.1554	10
12.7 mm, Antiair-craft Machine Gun, 1977	3	2.0	22	1.1718	1.1707	0.0881	4.3435	4.3415	0.0475	18.5030	18.4878	0.0821	11
MG42 7.92 mm, Machine Gun, Germany	3	1.4	18	0.8170	0.8165	0.0619	3.0339	3.0329	0.0333	15.9167	15.9074	0.0587	12.86
12.7 mm, Antiair-craft Machine Gun, 1954	3	1.6	21	0.9332	0.9327	0.0578	3.4665	3.4654	0.0311	15.6012	15.5926	0.0550	13.13
H-37, Aircraft Automatic Gun	4	2.3	21	1.7466	1.7321	0.8304	5.7932	5.7642	0.5007	27.5909	27.3952	0.7091	9.13
20 mm, Antiair-craft Automatic Gun	4	2.0	20	1.4982	1.4896	0.5699	4.9963	4.9793	0.3418	25.2047	25.0788	0.4997	10
AM-23, Aircraft Automatic Gun	4	1.8	18	1.3484	1.3407	0.5699	4.4967	4.4813	0.3418	25.2047	25.0788	0.4997	10



(a) (b)

Fig. 7. Relationships between (a) β and S/d (b) S/dN and ε (%) (The relative error in Fig. 7(b) is that of the β of the two models. The other two relative errors are consistent with it.).

The relationship between β and S/d is almost inverse. This is consistent with the previous description. When the S/dN range is in (1, 2), the relative error increases very quickly with the decrease of S/dN . In order to obtain an acceptable relative error, the S/d ratio must be two times larger than the number of wires in the strand, that is, $S/d > 2N$. In such cases, the approximate mathematical model can be used for convenience. Otherwise (if the S/d ratio is not two times larger than the number of wires in the strand), the approximate mathematical model cannot be applied, owing to the high relative errors. In order to verify our analysis, the values of some other strands were calculated (see Table 2).

Table 2 confirms our analytical assumptions. Specifically, the twist angle β is close to 30 degrees when the S/d ratio is more or less equal to $2N$. And when the S/d ratio is very close to N , the relative errors are extraordinarily high.

Usually, the twist angle is about 25~30 degrees in order to avoid higher stresses in the inner sides of the wires. Under these circumstances, the approximate mathematical model can be used, because the relative errors are acceptable. However, the application range of stranded-wire helical springs is very wide. In some cases for example, when springs must have higher stiffness and absorb higher vibrations, the twist angles are required to be around 40 degrees or even to exceed 40 degrees. In these situations, it is not appropriate to calculate the twist angle and diameter of strands with the approximate mathematical model.

4. Applications

For convenient design of stranded-wire helical springs, application software was developed for direct calculation of the twist angle and diameter of strands formed of arbitrary wires, according to the precise mathematical model. The software was designed in order to solve two problems: the ineffectiveness of the approximate mathematical model in some cases; the inapplicableness of the existing spring handbooks when

Table 2. Some other strands for stranded-wire helical springs.

N	d/mm	S/mm	r/mm			d_c /mm			β /°			$\frac{S}{d}$
			I	II	ε (%)	I	II	ε (%)	I	II	ε (%)	
3	2.0	10	1.2944	1.2391	4.2738	4.5889	4.4782	2.4111	39.1219	37.9026	3.1165	5
6	2.0	25	2.2379	2.2215	0.7307	6.4758	6.4431	0.5050	29.3552	29.1760	0.6104	12.5
6	1.0	8.5	1.3637	1.3017	4.5470	3.7274	3.6034	3.3271	45.2293	43.8964	2.9470	8.5
8	2.2	19	7.4207	6.0098	19.0130	17.0415	14.2197	16.5585	67.8292	63.2900	6.6920	8.64
8	2.1	23	3.9361	3.8033	3.3733	9.9721	9.7066	2.6630	47.0771	46.0955	2.0849	10.95
8	2.1	36	3.0703	3.0595	0.3502	8.2405	8.2190	0.2609	28.1852	28.1016	0.2966	17.14

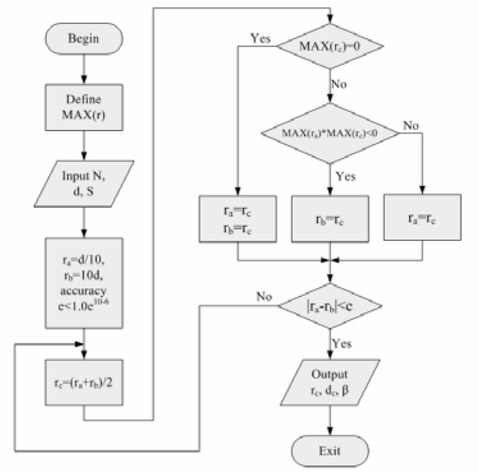


Fig. 8. Program flowchart of software.

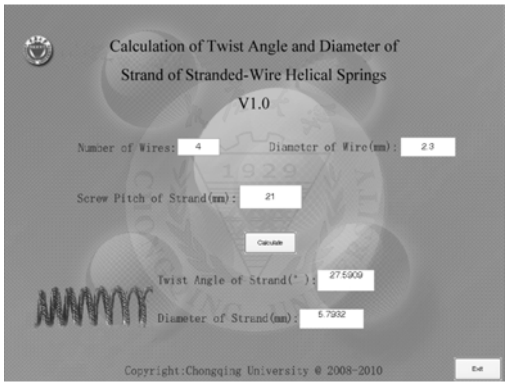


Fig. 9. Calculation of twist angle and diameter of strands.

the number of wires exceeds four. A program flowchart of the software is provided in Fig. 8. With the software, the precise twist angle and diameter of strands can be calculated directly, simply by inputting the number of wires, the diameter of wires and the screw pitch of strands (see Fig. 9). Furthermore, the software includes a numerical reference standard for the manufacture of stranded-wire helical springs.

5. Conclusions

This paper proposes two models for calculation of the twist angle and diameter of strands formed of arbitrary wires. For strands with the same number of wires, the larger the S/d ratio is, the smaller are the relative errors of the two models. Only when the S/d ratio is two times larger than N, the results obtained with the approximate mathematical model are acceptable. In other cases, this model cannot be employed.

Additionally, application software was developed for precise direct calculation of the twist angle and strand diameter. Significantly, with the software, there is no need for researchers to look up data in handbooks. More specifically, the software overcomes the problems encountered with some applications of the approximate mathematical model and renders irrelevant the limitation that existing spring handbooks cannot be referenced when the number of wires exceeds four.

The next research step will be to consider the internal relations between springback and springs and to optimize the twist angle and strand diameter values.

Acknowledgement

This research was supported by the National Natural Science Funds for Distinguished Young Scholars (No. 50925518), the National Science Foundation of China (No. 50775226), the National Key Project for High-end CNC Machine Tools and Basic Manufacturing Equipment (No. 2009ZX04001-081 and No. 2009ZX04011-041), the Key Project of the Chinese Ministry of Education (No. 109129), the Chongqing Key Scientific and Technological Project (No. CSTC2009AC3049), the Chongqing University Postgraduates Science and Innovation Fund (No. 200911A1A0020318) and the Innovative Talent Training Project, the Third Stage of '211 Project', Chongqing University (No. S-09106).

Nomenclature

β : Twist angle of strand
d_c : Diameter of strand
d : Diameter of wire
S : Screw pitch of strand
N : Total number of wires in strand
α : Helix angle of strand

r : Center distribution radius (i.e. distance between center point of wire and center point of strand)

ϕ, θ, φ : Euler angles

References

[1] G. A. Costello and J. W. Phillips, Static Response of Stranded-Wire Helical Springs, *International Journal of Mechanical Sciences*, 21 (1979) 171-178.

[2] H. H. Clark, Stranded-Wire Helical Springs, *Spring Design and Application*, McGraw-Hill Book Company, New York, USA, (1961) 92-96.

[3] J. W. Phillips and G. A. Costello, General Axial Response of Stranded-Wire Helical Springs, *International Journal of Non-Linear Mechanics*, 14 (1979) 247-257.

[4] H. Carlson, *Spring Manufacturing Handbook*, Marcel Dekker, New York, USA, (1982) 141.

[5] J. J. Min and S. L. Wang, Analysis on Dynamic Calculation of Stranded-Wire Helical Spring, *Chinese Journal of Mechanical Engineering*, 43 (3) (2007) 199-203.

[6] H. Carlson, *Spring Designer's Handbook*, Marcel Dekker, New York, USA, (1978) 232.

[7] D. W. Yu, The Dynamic Stress of Stranded-Wire Helical Spring and Its Useful Life, *Journal of Nanjing University of Science and Technology*, 75 (3) (1994) 24-29.

[8] The Spring Research Association, *Springs Materials/Design/Manufacture*, J. W. Northend, Sheffield, UK, (1968).

[9] Y. H. Zhang, H. H. Liu and D. C. Wang, *Spring Handbook*, Mechanical Industry Press, Beijing, PRC, (1999) 312-329.

[10] M. L. Wang, Analytic Calculation of Multi-wire Helical Compression Springs With Multi-center-wires, *Acta Armamentar II*, (1995) 60-65.

[11] D. W. Zhang, *Handbook of Spring Design, New Technology of Manufacturing Process and Quality Control*, Chinese Science and Culture Audio-video Publishing House, Beijing, PRC, (2003) 182-208.

[12] S. Sathikh, M. B. K. Moorthy and M. Krishnan, A Symmetric Linear Elastic Model for Helical Wire Strands under Axisymmetric Loads, *Journal of Strain Analysis*, 31 (5) (1996) 389-399.

[13] S. L. Wang, J. Zhou and L. Kang, Dynamic Tension of Stranded-Wire Helical Spring during Reeling, *Chinese Journal of Mechanical Engineering*, 44 (6) (2008) 36-42.

[14] D. R. Wu, *An Introduction to Space Analytic Geometry*, Higher Education Press, Beijing, PRC, (1989) 207-211.

Shi-Long Wang received his B.S., M.S. and Ph.D. degrees in Mechanical Engineering from Chongqing University, China, in 1988, 1991 and 1995, respectively. Dr. Wang currently is a Professor at the School of Mechanical Engineering, Chongqing University, Chongqing, China. He serves as a director of the *Chinese Journal of Mechanical Engineering*. Dr. Wang's research interests include manufacturing automation, computer integrated manufacturing and enterprises informatization.

Song Lei received his B.S. degree in Mechanical Engineering from Chongqing University, China, in 2005. He received his M.S. degree in Robotics and Mechatronics from Sant'Anna University, Italy, in 2007. Lei currently is a Ph.D. candidate at the School of Mechanical Engineering, Chongqing University, Chongqing, China.

Jie Zhou received his B.S. degree in Mechanical Engineering and his M.S. degree in Software Engineering from Chongqing University in 1988 and 2007, respectively. Zhou currently is an Associate Professor at the School of Mechanical Engineering, Chongqing University, Chongqing, China. Zhou's research interests include manufacturing automation and mechatronics.

Hong Xiao received her B.S. degree in Mechanical Engineering from Yantai University, China, in 2001. She received her M.S. degree in Mechanical Engineering from Chongqing University in 2007. Xiao is currently a Ph.D. candidate at the School of Mechanical Engineering, Chongqing University, Chongqing, China.